

Exercise 15

Use the definition of continuity and the properties of limits to show that the function is continuous on the given interval.

$$f(x) = x + \sqrt{x - 4}, \quad [4, \infty)$$

Solution

Assuming that $4 \leq a < \infty$, calculate the limit of $f(x)$ as x approaches a using the limit laws.

$$\begin{aligned} \lim_{x \rightarrow a} f(x) &= \lim_{x \rightarrow a} (x + \sqrt{x - 4}) \\ &= \lim_{x \rightarrow a} x + \lim_{x \rightarrow a} \sqrt{x - 4} \\ &= a + \sqrt{\lim_{x \rightarrow a} (x - 4)} \\ &= a + \sqrt{\lim_{x \rightarrow a} x - \lim_{x \rightarrow a} 4} \\ &= a + \sqrt{a - 4} \\ &= f(a) \end{aligned}$$

Therefore, $f(x) = x + \sqrt{x - 4}$ is continuous on the interval $[4, \infty)$.